



Benchmarking of optimization methods for topology optimization problems

Rojas Labanda, Susana; Stolpe, Mathias

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Why?

- Asses general purpose 2nd order optimization methods in topology optimization problems.

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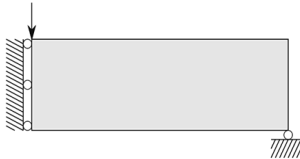
Main results from the Benchmarking

- GCMMA outperforms MMA.
- GCMMA and MMA tend to obtain a design with large KKT error.
- The performance of GCMMA and MMA do not highlight respect to other solvers.
- The interior-point solver IPOPT, when the exact Hessian is used (IPOPT SAND), produces the best designs using few number of iterations
- IPOPT SAND is the most robust solver in the study.
- The SAND formulation requires lot of memory and computational time.



Topology optimization problems

- **Goal:** Obtain optimal design of a structure with given loads.



- **Model as an optimization problem**

$$\begin{aligned} & \underset{x}{\text{minimize}} && f(x) \\ & \text{subject to} && g(x) \leq 0 \\ & && h(x) = 0. \end{aligned}$$

Topology optimization formulations

SAND formulation:

- Minimum compliance

$$\begin{aligned} &\underset{\mathbf{t}, \mathbf{u}}{\text{minimize}} && \mathbf{f}^T \mathbf{u} \\ &\text{subject to} && \mathbf{a}^T \mathbf{t} \leq V \\ &&& \mathbf{K}(\mathbf{t}) \mathbf{u} - \mathbf{f} = \mathbf{0} \\ &&& \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{aligned}$$

- Minimum volume

$$\begin{aligned} &\underset{\mathbf{t}, \mathbf{u}}{\text{minimize}} && \mathbf{a}^T \mathbf{t} \\ &\text{subject to} && \mathbf{f}^T \mathbf{u} \leq C \\ &&& \mathbf{K}(\mathbf{t}) \mathbf{u} - \mathbf{f} = \mathbf{0} \\ &&& \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{aligned}$$

- Compliant mechanism design

$$\begin{aligned} &\underset{\mathbf{t}, \mathbf{u}}{\text{minimize}} && \mathbf{l}^T \mathbf{u} \\ &\text{subject to} && \mathbf{a}^T \mathbf{t} \leq V \\ &&& \mathbf{K}(\mathbf{t}) \mathbf{u} - \mathbf{f} = \mathbf{0} \\ &&& \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{aligned}$$

- $\mathbf{f} \in \mathbb{R}^d$ the force vector.
- $\mathbf{a} \in \mathbb{R}^n$ the volume vector.
- $V > 0$ is the upper volume fraction.
- $C > 0$ the upper bound of the compliance.
- $\mathbf{l} \in \mathbb{R}^d$ vector that indicates the output displacement.

Topology optimization formulations

NESTED formulation:

- Minimum compliance
- Minimum volume
- Compliant mechanism design

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{u}^T(\mathbf{t})\mathbf{K}(\mathbf{t})\mathbf{u}(\mathbf{t}) \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{a}^T\mathbf{t} \\ \text{subject to} & \mathbf{u}^T(\mathbf{t})\mathbf{K}(\mathbf{t})\mathbf{u}(\mathbf{t}) \leq C \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{l}^T\mathbf{u}(\mathbf{t}) \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- $\mathbf{u}(\mathbf{t}) = \mathbf{K}^{-1}(\mathbf{t})\mathbf{f}$.
- $\mathbf{f} \in \mathbb{R}^d$ the force vector.
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Considerations on the problem formulation

- Use only one external static load.
- Linear elasticity in the equilibrium equation.
- Assume $\mathbf{K}(\mathbf{t}) \succ 0$ to avoid ill-conditioning.
- Use continuous density variables.
- Use SIMP penalization and a density filter.

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Bourdin, B. Filters in topology optimization. *International Journal for Numerical Methods in Engineering*, 50(9):2143–2158, 2001.

Optimization methods

Topology
optimization
problem



- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.

Andreassen, E and Clausen, A and Schevenels, M and Lazarov, B. S and Sigmund, O. Efficient topology optimization in MATLAB using 88 lines of code. *Structural and Multidisciplinary Optimization*, 43(1): 1–16, 2011.

Svanberg, K. The method of moving asymptotes a new method for structural optimization. *International Journal for Numerical Methods in Engineering*, 24(2): 359–373. 1987.

Svanberg, K. A class of globally convergent optimization methods based on conservative convex separable approximations. *SIAM Journal on Optimization*, 12(2): 555-573, 2002.

Optimization methods

Topology
optimization
problem



non-linear
problem

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.
- **FMINCON**: Interior-point MATLAB. Use exact Hessian.
- **SNOPT**: Sequential quadratic programming. BFGS approximations.
- **IPOPT**: Interior-point software. Exact Hessian in the SAND formulation, BFGS in the NESTED formulation.

Gill, P. E and Murray, W and Saunders, M. A. SNOPT: An SQP Algorithm for Large -Scale Constrained Optimization. *SIAM Journal on Optimization*, 47(4):99–131, 2005.

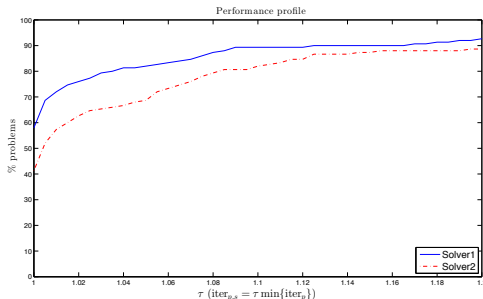
Wächter, A and Biegler, L. T. On the implementation of an interior point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, 2006.

Benchmarking in topology optimization

- **How?** Using performance profiles.
 - Evaluate the cumulative ratio for a performance metric.
 - Represent for each solver, the percentage of instances that achieve a criterion for different ratio values.

$$\rho_s(\tau) = \frac{1}{n} \text{size}\{p \in P : r_{p,s} \leq \tau\},$$

$$r_{p,s} = \frac{\text{iter}_{p,s}}{\min\{\text{iter}_{p,s} : s \in S\}}.$$



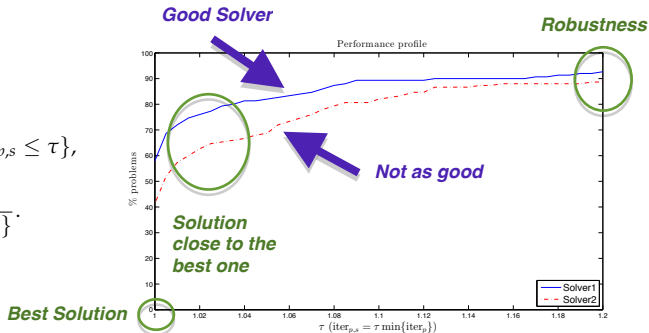
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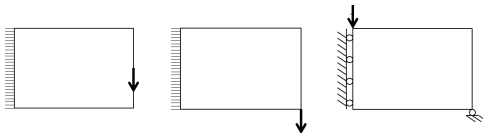


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Benchmark set of topology optimization problems

Minimum compliance /minimum volume

- Michell, Cantilever and MBB domains, respectively.

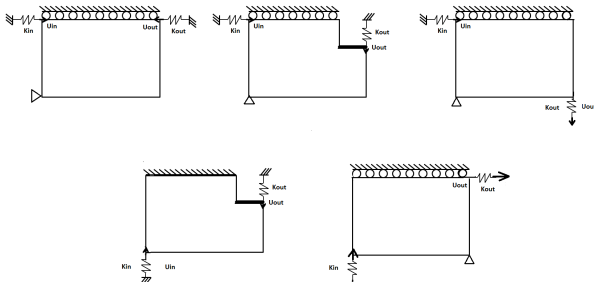


- Length ratio: Michell: 1×1 , 2×1 , and 3×1 . Cantilever: 2×1 , and 4×1 . MBB: 1×2 , 1×4 , 2×1 , and 4×1 .
- Discretization: 20, 40, 60, 80, 100 elements per ratio.
- Volume constraint: 0.1 – 0.5.
- Compliance constraint: 1, 1.25, $1.5 \times C$. Where $C = \mathbf{f}^T \mathbf{K}^{-1}(\mathbf{t}_0) \mathbf{f}$.
- Total Problems Compliance: 225.
- Total Problems Volume: 135.

Benchmark set of topology optimization problems

Compliant mechanism design

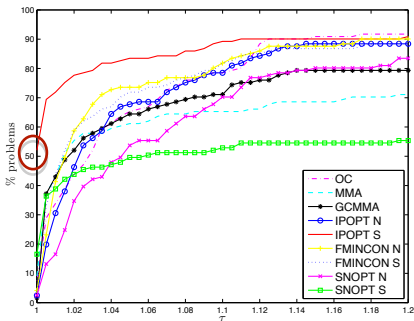
- Force inverter, Compliant gripper, Amplifier, Compliant lever, and Crimper domain examples, respectively.



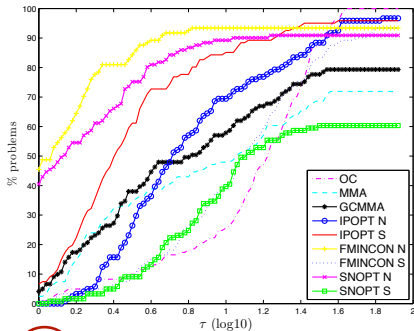
- Length ratio: 1×1 and 2×1 .
- Volume constraint: 0.2 – 0.4
- Discretization: 20, 40, 60, 80, 100 elements per ratio.
- Total Problems Mechanism Design: 150.**

Performance profiles for minimum compliance problems

Objective function value



Number of iterations

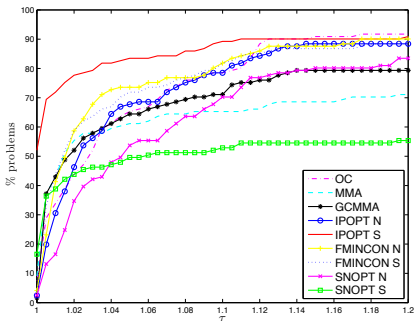


Performance profiles in a reduce test set of 121 instances.

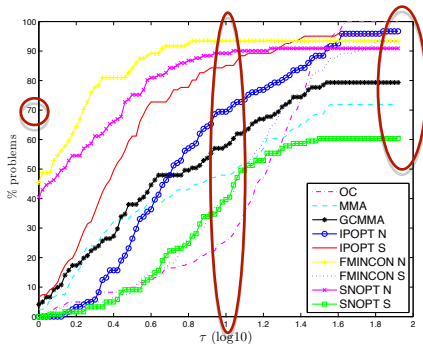
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for minimum compliance problems

Objective function value



Number of iterations

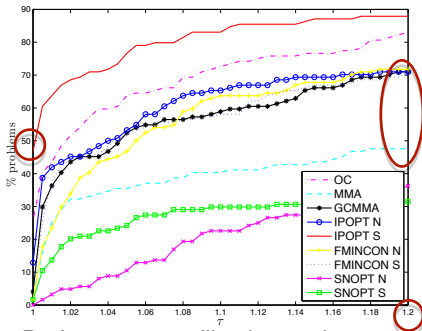


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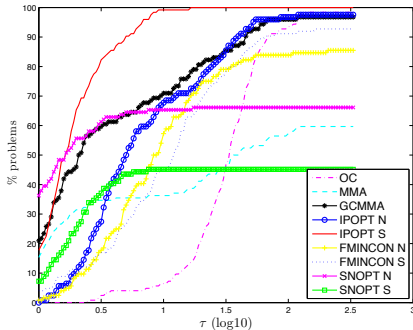
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for compliant mechanism design problems

Objective function value



Number of iterations

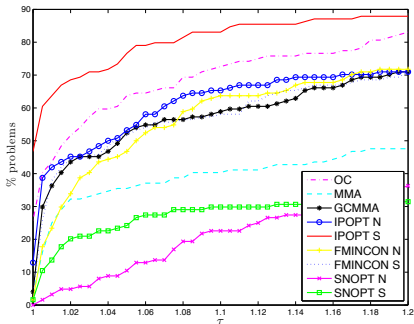


Performance profiles in a reduce test set of 124 instances.

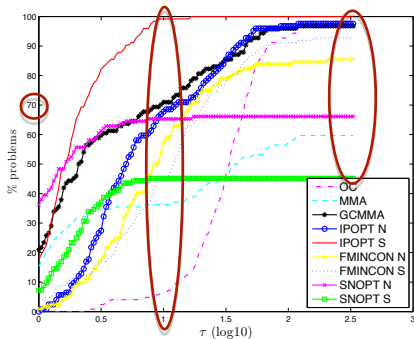
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for compliant mechanism design problems

Objective function value



Number of iterations

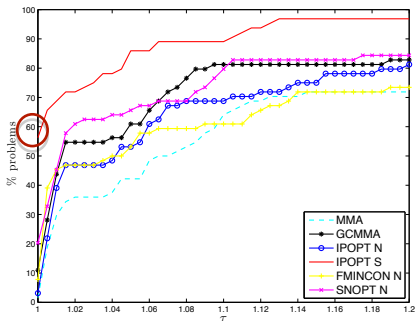


Performance profiles in a reduce test set of 124 instances.

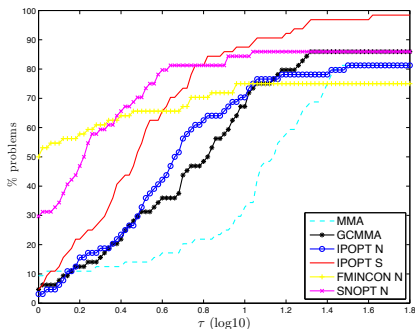
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for minimum volume problems

Objective function value



Number of iterations

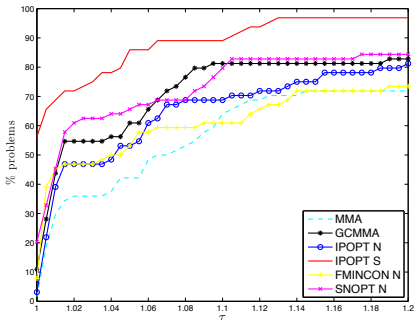


Performance profiles in a reduce test set of 64 instances.

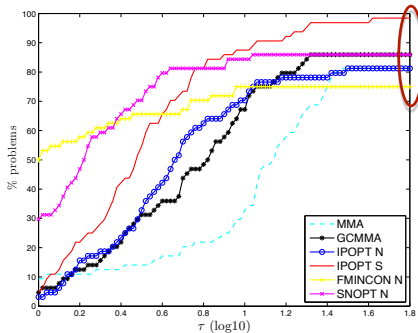
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for minimum volume problems

Objective function value



Number of iterations



Performance profiles in a reduce test set of 64 instances.

Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Conclusions

- **Important contributions.**
 - Develop a large topology optimization test set.
 - Introduction to performance profiles in topology optimization.
 - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

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 - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.
- **What is missing?**
 - Large-scale problems, 3D domains, advance elements,...
 - Other regularization schemes.
 - Different formulations: Displacement constraint, stress constraints,...
 - More optimization solvers.
 - ...

Conclusions

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- Develop a large topology optimization test set.
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- First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

- **What is missing?**

- Large-scale problems, 3D domains, advance elements,...
- Other regularization schemes.
- Different formulations: Displacement constraint, stress constraints,...
- More optimization solvers.
- ...

- **What can we conclude from the performance profiles?**

- GCMMA outperforms MMA.
- GCMMA and MMA tend to obtain a design with large KKT error.
- IPOPT-S produces better designs using few number of iterations
- IPOPT-S is the most robust solver in the study.
- The SAND formulation requires lot of memory and computational time.





THANK YOU !!!

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